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Conference Proceedings

SEOUL 2014
BRIDGES

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Preface

Welcome to Bridges Seoul 2014, the 17th annual Bridges conference! This year we are thrilled to build our longest bridge yet, making our way to Asia for the first time after our many travels across North America and Europe. We look forward to the opportunity to interact with a new community of participants. We hope also to foster excitement about art and mathematics by engaging with the Korean public in cooperation with our hosts, the Gwacheon National Science Museum.

The Bridges board of directors is grateful to Professor Ingrid Daubechies, President of the International Mathematical Union, for suggesting that we hold Bridges in Seoul as a satellite conference to the 2014 International Congress of Mathematicians (Seoul ICM 2014), and for initiating the contact between Bridges, the ICM, and the Gwacheon National Science Museum. Based on several site visits and a series of meetings with the Gwacheon Museum Board of Directors and ICM organizers, we have assembled an international committee that has worked hard to make this conference a reality.

Gwacheon National Science Museum, which opened its doors in 2008, is the largest science museum in Asia, and one of the largest in the world. Through its highly interactive exhibits and permanent collections, it strives to communicate scientific knowledge to a broad audience in an accessible way. Bridges is naturally aligned with this point of view—from the beginning, the conference has explored how to use art to talk about mathematics and aid the acquisition and retention of mathematical ideas. Conversely, we are pleased to observe that the museum's vision statement demands that exhibitions be founded upon STEAM (Science, Technology, Engineering, Art, Mathematics), and not just STEM.

The Bridges Organization's educational goals are clearer than ever this year, as we simultaneously inaugurate MoSAIC (Mathematics of Science, Art, Industry, Culture), a series of mathematical art mini-conferences sponsored by the Mathematical Sciences Research Institute (MSRI). MSRI is one of the world's preeminent organizations for collaborative research. MoSAIC events can be organized anywhere, and funding is available for guest speakers and hands-on workshops. More information on MoSAIC can be found at `www.mosaicmathart.org`.

Mathematics, art, and science all date far back to the prehistory of mankind and have in common that they involve abstractions from observations of nature. Various types of patterns and structures naturally arise in these three fields. It is a central theme of Bridges conference papers to elucidate and depict such structures, so this year's setting in the Science Museum gives us a particularly appropriate environment in which to ponder the roots of the structures found in math, art, and science.

This year's Program Committee Chair is Gary Greenfield, with George Hart and Reza Sarhangi serving as co-editors of the proceedings. Under Gary's direction, a committee comprising more than forty experts from around the world provided extensive and rigorous reviews to submissions in three categories—regular papers, short papers, and workshop papers—and offered further feedback and advice to authors of accepted papers in order to improve their final versions. This process ultimately yielded the 37 regular papers, 44 short papers, and 8 workshop papers that are included in this volume. The editors would like to acknowledge the dedicated reviewing efforts of Mara Alagic, Bob Bosch, Paul Gailiunas, Craig S. Kaplan, Douglas M. McKenna, and Carlo Séquin who generously provided extra support. We thank all the authors, program committee members and other volunteers for their careful work.

An exhibition of mathematical art has been an annual feature of Bridges since 2001, and well over 100 artists contributed to this year's art exhibition. The list of contributors includes several newcomers from Japan,

South Korea, and China, as well as artists from North and South America, Europe, Africa, and Australia. A wide variety of artistic media are represented in the exhibition, including 2D and 3D digital prints, painting, beadwork, ceramics, wood, metal, quilting, and paper folding. Artists drew inspiration from the mathematics of fractals, polyhedra, non-Euclidean and four-dimensional geometry, tiling, knot theory, magic squares, and more. This year Katie McCallum and Robert Fathauer served as co-curators of the exhibition, and were joined by Anne Burns, Nat Friedman and Chaesoon Kwon to make up the jury. The print catalog was edited by Conan Chadbourne, Robert Fathauer, and Katie McCallum.

Once again, we are pleased to welcome you to this year's conference. We hope that you find insight and inspiration in the papers in this volume, in the diverse events that make up Bridges, and in our wonderful location.

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