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An Analysis of the Bricard Linkages

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Abstract
Mobile overconstrained loops are now familiar to linkage kinematicians, but 50 years ago they posed a puzzle to the few who studied the motions of jointed rigid bodies. Ranking high in significance among the "paradoxical" linkages is the group due to Bricard, from the first quarter of this century. Unfortunately, these 6-revolute loops, of considerable interest to line geometricians, have been subject to a great deal of misunderstanding, and have never been delineated by appropriate sets of independent closure equations. This paper is an overdue attempt to remedy the situation.

Introduction

AMONG THE MANY mobile overconstrained linkages now known, the 6-revolute loops seem to hold the greatest fascination for kinematicians, particularly, and understandably, those who have a geometrically oriented viewpoint. Of the 6-R chains, the most remarkable, and probably the least generally known, are the three types which Bricard[1] discovered as a result of his investigation into mobile octahedra. We shall refer to them here as the "octahedral" linkages, although Goldberg's[2] term, "6-plate" linkages, is undeniably the most accurate brief description.

For someone who sets out to define a linkage by means of interparametric constraints and an independent set of closure equations, the difficulties are considerably magnified as the number of links increases. At this stage, we still have not quite completed[3] defining all mobile four-bar loops. It is not surprising, then, that none of the five distinct Bricard linkages has been algebraically delineated. The Schatz linkage[4], which was inspired by a special Bricard loop, has been analysed by Brát[5], and the Myard and Goldberg loops have recently been treated[6].

Apart from the fact that the Bricard linkages present a considerable algebraic challenge, and that their solutions will complete the assault on the known loops of their general character, it is hoped that the following analysis will clear away the misconceptions concerning them and even, for many workers perhaps, bring them out of the kinematic unknown. In addition, the results should be of value to those involved in a current resurgence of interest in line geometry. Bottema has also expressed curiosity about the relative motion between links of the octahedral chains in the closing paragraph of [7], from which paper possible applications of the linkages to structural organic chemistry may be inferred.

In the analysis to follow, we shall use a terminology which is well known and most clearly defined by an illustration, such as Fig. 1. For brevity, we shall write *cosine* as c and *sine* as s. There will be other special conventions for the octahedral linkages which will be described at the appropriate places. We shall refer to the 6-bar closure eqns (A6.1–A6.12) in the form given in the Appendix. The reader is reminded in this context that eqns (A6.1–A6.9) are "rotational" equations, derivable from a spherical indicatrix and, as they stand, representing at most three independent equations. Equations (A6.10–A6.12) are "translational" equations, generally independent and derivable, in principle, as dual relationships from the rotational set. We shall also refer to certain 5-bar closure eqns (A5.9, A5.12) and to a 4-bar eqn (A4.9), as given in the Appendix.

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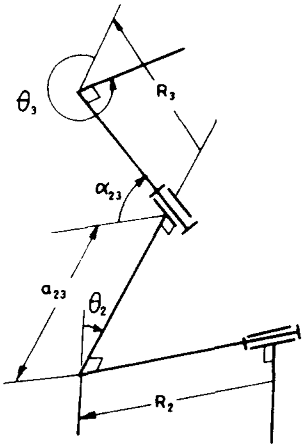

Figure 1.

The present work is of a style somewhat similar to that of [6], to the extent that the mobility of each linkage treated is assumed at the outset. In each case a fairly arbitrary, but economical, set of independent closure equations will be developed. The work will not follow the more comprehensive, searching approach for existence criteria adopted in [3, 8], for example.

The Linkages

Although, in chronological order of publication, the octahedral linkages were the first 6-*R* loops to be determined by Bricard, it will be clear why it is more appropriate to first consider his other discoveries, assuming that he was primarily responsible for all of them. At this point I wish, in passing, to attest to my great respect for Bricard's work and my admiration of his mathematical power. I do so because it will be relevant in places to point out his apparent lack of rigour or knowledge, and it is conceivable that Bricard's own minor failings were in part responsible for the confusion which abounds concerning his linkages.

In a summary[9] of "mécanismes paradoxaux" containing only turning pairs, Bricard lists three distinct 6-bar loops as well as his octahedral linkages. He has previously pointed out that, for mobility, all six joint axes must belong to a linear complex, but gives a reason for mobility of each of the first three linkages separately.

1. *The general line-symmetric case*

The 6-*R* line-symmetric loop in its greatest generality is illustrated by the model shown in Fig. 2. It is to be noted, in particular, that offsets are non-zero and skew angles between adjacent joints are not necessarily rightangles. It is more common to observe special forms of this linkage, such as the model shown in Fig. 3. I believe that Bricard's explanation of this loop's mobility is facile, and the reader is referred to Waldron's[8, 10] screw system analysis of the 6-*H* line-symmetric chain for a satisfactory treatment.

It is the easiest of the Bricard loops to analyse algebraically. One has immediately the parametric constraints

$$a_{12} = a_{45} \qquad a_{23} = a_{56} \qquad a_{34} = a_{61}$$

$$\alpha_{12} = \alpha_{45} \qquad \alpha_{23} = \alpha_{56} \qquad \alpha_{34} = \alpha_{61}$$

$$R_1 = R_4 \qquad R_2 = R_5 \qquad R_3 = R_6$$

and the independent closure equations

$$\theta_1 = \theta_4 \tag{1.1}$$

$$\theta_2 = \theta_5 \tag{1.2}$$

$$\theta_3 = \theta_6. \tag{1.3}$$

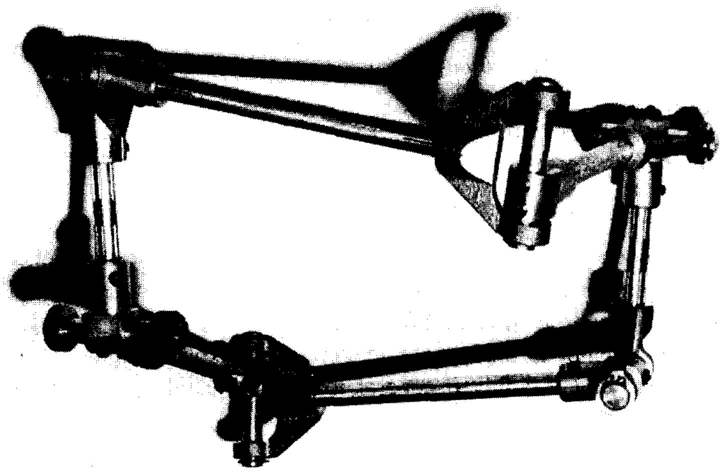

Figure 2.

Substitution of the above relationships between skew angles and joint rotations into eqns (A6.1–A6.9) results in some of the equations being identically satisfied. Of the others, it can be shown that there is only one independent rotational equation, such as

$$\begin{aligned}
&c\alpha_{34}(s\theta_2 c\theta_3 s\alpha_{12} + c\theta_2 s\theta_3 s\alpha_{12} c\alpha_{23} + s\theta_3 c\alpha_{12} s\alpha_{23})\\
&+ s\alpha_{34}(s\theta_1 c\theta_2 c\theta_3 + c\theta_1 s\theta_2 c\theta_3 c\alpha_{12} - s\theta_1 s\theta_2 s\theta_3 c\alpha_{23}\\
&+ c\theta_1 c\theta_2 s\theta_3 c\alpha_{12} c\alpha_{23}\\
&- c\theta_1 s\theta_3 s\alpha_{12} s\alpha_{23})\\
&= c\theta_1 s\theta_2 s\alpha_{23} + s\theta_1 c\theta_2 c\alpha_{12} s\alpha_{23} + s\theta_1 s\alpha_{12} c\alpha_{23}.
\end{aligned} \tag{1.4}$$

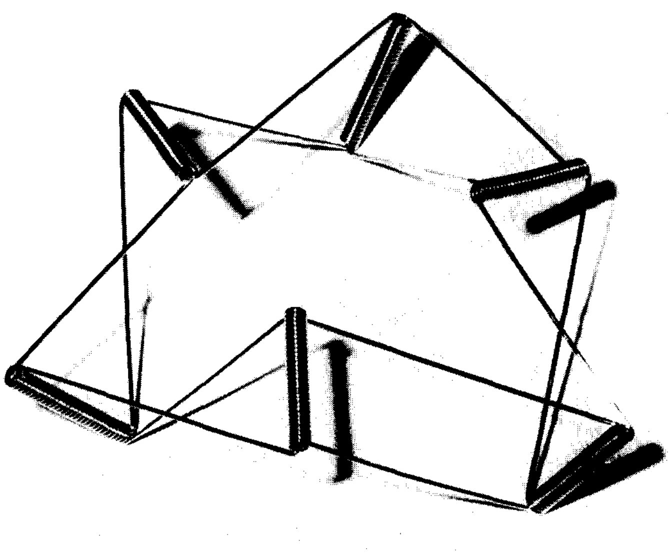

Figure 3.

It can similarly be shown that only one of the translational eqns (A6.10–A6.12) is independent, and we choose

$$\left.\begin{aligned}&a_{12}(c\theta_1 + c\theta_2 c\theta_3 - s\theta_2 s\theta_3 c\alpha_{23}) + R_1(s\theta_2 c\theta_3 s\alpha_{12}\\&+ c\theta_2 s\theta_3 s\alpha_{12} c\alpha_{23} + s\theta_3 c\alpha_{12} s\alpha_{23}) + a_{23}(c\theta_1 c\theta_2 - s\theta_1 s\theta_2 c\alpha_{12}\\&+ c\theta_3) + R_2(s\theta_1 s\alpha_{12} + s\theta_3 s\alpha_{23}) + a_{34}(1 + c\theta_1 c\theta_2 c\theta_3\\&- s\theta_1 s\theta_2 c\theta_3 c\alpha_{12} - c\theta_1 s\theta_2 s\theta_3 c\alpha_{23} - s\theta_1 c\theta_2 s\theta_3 c\alpha_{12} c\alpha_{23}\\&+ s\theta_1 s\theta_3 s\alpha_{12} s\alpha_{23}) + R_3(c\theta_1 s\theta_2 s\alpha_{23}\\&+ s\theta_1 c\theta_2 c\alpha_{12} s\alpha_{23} + s\theta_1 s\alpha_{12} c\alpha_{23}) = 0.\end{aligned}\right\} \quad (1.5)$$

Equations (1.1–1.5) are a set of independent closure equations for the general line-symmetric 6-R loop. As already suggested, alternative equations are available and, in some cases, one of them will be necessary to locate the appropriate quadrant for a joint angle which might be sought for a particular configuration. No attempt is made here to determine explicit input–output relationships for all joint angles. These remarks apply, in general, to each of the linkages examined in this paper.

2. *The general plane-symmetric case*

A model of a general plane-symmetric 6-R linkage is shown in Fig. 4. More common examples of special cases are illustrated by the models of Fig. 5. As for the line-symmetric loop, so for the general form of this linkage, offsets are non-zero and skew angles not rightangles. Again, readers are directed away from Bricard's explanation of the loop's mobility to that of Waldron[8, 10] for the corresponding 6-H chain.

For the general Bricard loop, we have the dimensional conditions, say,

$$a_{61} = a_{12} \qquad a_{56} = a_{23} \qquad a_{45} = a_{34}$$

$$\alpha_{61} + \alpha_{12} = \pi \qquad \alpha_{56} + \alpha_{23} = \pi \qquad \alpha_{45} + \alpha_{34} = \pi$$

$$R_1 = R_4 = 0 \qquad R_6 = R_2 \qquad R_5 = R_3$$

and two independent closure equations

$$\theta_6 + \theta_2 = 2\pi \quad (2.1)$$

$$\theta_5 + \theta_3 = 2\pi. \quad (2.2)$$

We have thereby chosen joint axes 1 and 4 to lie in the linkage's plane of symmetry.

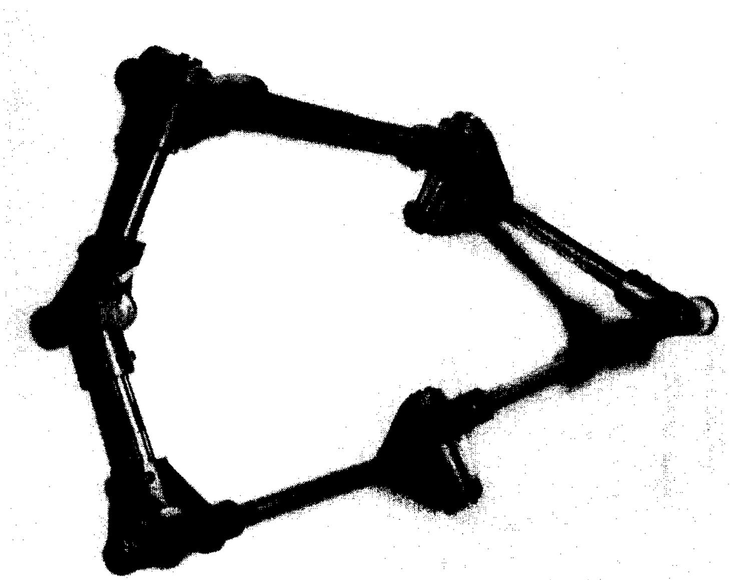

Figure 4.

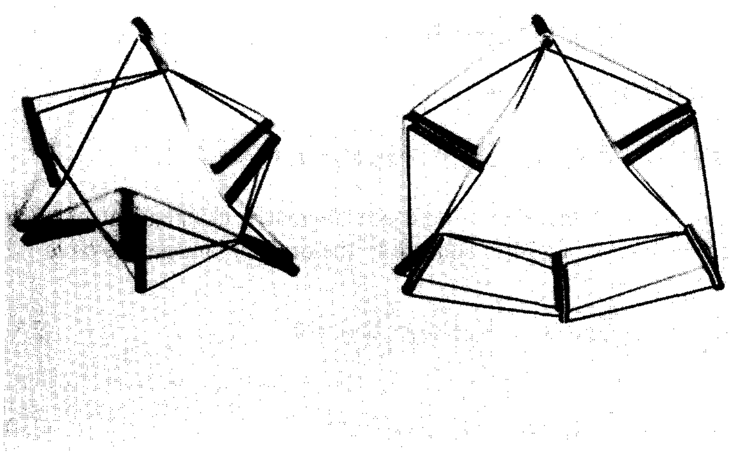

Figure 5.

Rather than attempt to simplify the general 6-bar equations to determine the remaining independent ones for this loop, we consider 'half' of the linkage by replacing joints 5 and 6 by a single revolute, joint 5′, which is directed at rightangles to the plane of symmetry and passes through the point of intersection of joints 1 and 4. In order for the substitute 5-bar loop to be mobile, joints 1 and 4 are made cylindric and renamed 1′ and 4′, respectively. As is clear from Fig. 6, the new linkage will be subject to the dimensional constraints

$$a_{4'5'} = R_{5'} = a_{5'1'} = 0$$

$$\alpha_{4'5'} = \frac{\pi}{2} \qquad \alpha_{5'1'} = \frac{3\pi}{2}.$$

We may now use the 5-bar closure equations for this special C-R-R-C-R- loop to establish relations among $\theta_1 - \theta_4$ for the original linkage. We clearly must avoid involving $r_{4'}$, $\theta_{5'}$, $r_{1'}$ in any equations and, from Fig. 6, we see that

$$\theta_{4'} = \frac{\pi + \theta_4}{2} \quad \theta_{1'} = \frac{\pi + \theta_1}{2}.$$

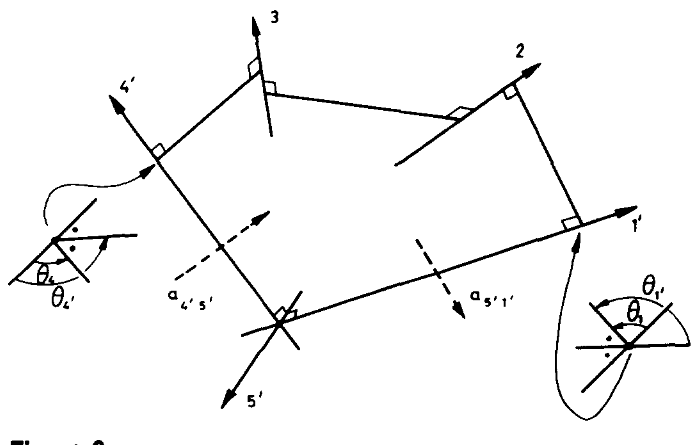

Figure 6.

Advancing the subscripts in (A5.9) by 4 and 2 yields, respectively, the two following equations.

$$c\theta_{1'}s\alpha_{12} = s\theta_3 s\theta_{4'}s\alpha_{23} - c\theta_3 c\theta_{4'}s\alpha_{23}c\alpha_{34} - c\theta_{4'}c\alpha_{23}s\alpha_{34}$$

$$c\theta_{4'}s\alpha_{34} = s\theta_{1'}s\theta s\alpha_{23} - c\theta_{1'}c\theta_2 s\alpha_{23}c\alpha_{12} - c\theta_{1'}c\alpha_{23}s\alpha_{12}$$

Advancing the subscripts in (A5.12) by 2 yields the result

$$-a_{23}(s\theta_{1'}c\theta_2 + c\theta_{1'}s\theta_2 c\alpha_{12}) + Rc\theta_{1'}s\alpha_{12} - a_{12}s\theta_{1'} + a_{34}s\theta_{4'} - R_3 c\theta_{4'}s\alpha_{34} = 0.$$

Alternative equations are available, but the above three are obviously independent of each other and of (2.1,2). Rewriting them now in a form suitable to the original linkage, we have

$$s\frac{\theta_1}{2}s\alpha_{12} + s\frac{\theta_4}{2}c\alpha_{23}s\alpha_{34} + c\theta_3 s\frac{\theta_4}{2}s\alpha_{23}s\alpha_{23}c\alpha_{34} + s\theta_3 c\frac{\theta_4}{2}s\alpha_{23} = 0 \tag{2.3}$$

$$s\frac{\theta_4}{2}s\alpha_{34} + s\frac{\theta_1}{2}c\alpha_{23}s\alpha_{12} + c\theta_2 s\frac{\theta_1}{2}s\alpha_{23}c\alpha_{12} + s\theta_2 c\frac{\theta_1}{2}s\alpha_{23} = 0 \tag{2.4}$$

$$a_{12}c\frac{\theta_1}{2} + a_{23}\left(c\frac{\theta_1}{2}c\theta_2 - s\frac{\theta_1}{2}s\theta_2 c\alpha_{12}\right) - a_{34}c\frac{\theta_4}{2} + R_2 s\frac{\theta_1}{2}s\alpha_{12} - R_3 s\frac{\theta_4}{2}s\alpha_{34} = 0 \tag{2.5}$$

Equations (2.1–2.5) are a set of independent closure equations for the general plane-symmetric 6-R linkage.

3. *The trihedral case*

This unique linkage, also described as "rectangular" and "two-point", was detailed and shown to be mobile in [9]. It is illustrated by the model of Fig. 7 and the schematic of Fig. 8 and has the geometrical properties, say,

$$\alpha_{12} = \alpha_{34} = \alpha_{56} = \frac{\pi}{2} \qquad \alpha_{23} = \alpha_{45} = \alpha_{61} = \frac{3\pi}{2}$$

$$R_i = 0, \quad \text{all } i$$

$$a_{12}^2 + a_{34}^2 + a_{56}^2 = a_{23}^2 + a_{45}^2 + a_{61}^2.$$

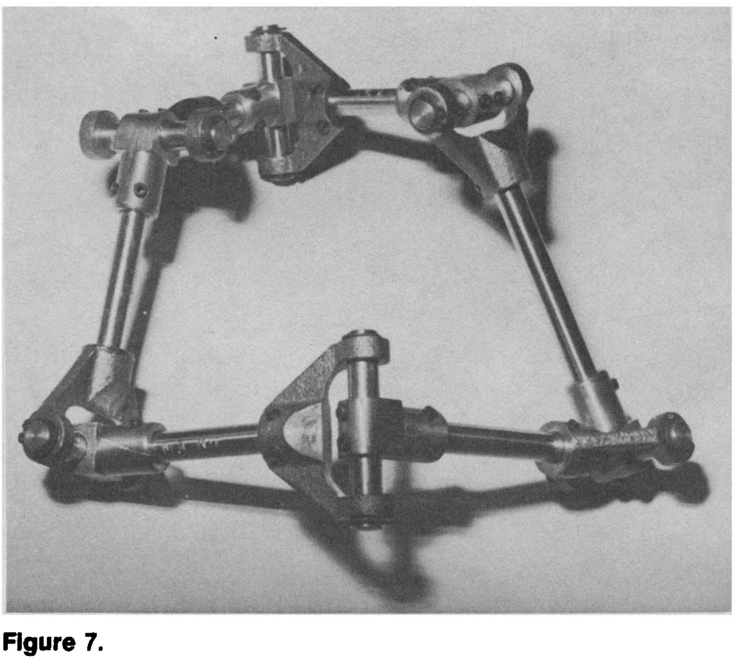

Figure 7.

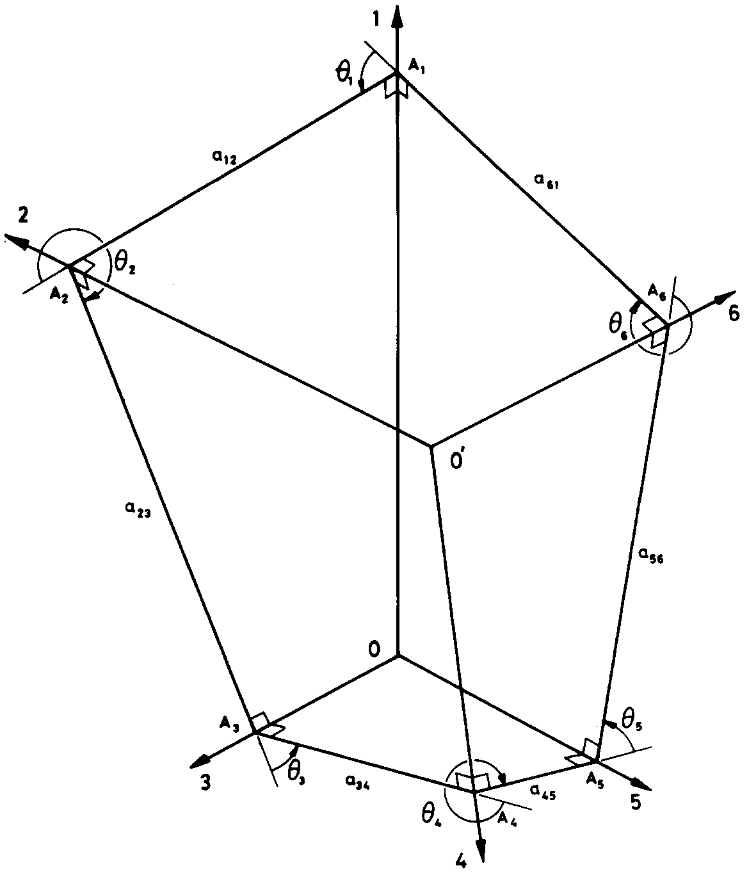

Figure 8.

Screw system theory is eminently suitable for demonstrating the full-cycle mobility of this chain. Because each set of three alternate joint axes is concurrent, all six axes pass through a single line defined by the two points. The six axes therefore lie permanently on a special linear complex, and the loop is mobile. The line through which they pass contains the single screw (of zero pitch) which is reciprocal to their 5-system.

Four independent closure equations are readily obtained from Fig. 8. Some elementary trigonometry applied to face $A_6A_1A_2O'$ of the hexahedron depicted establishes that

$$A_2O' = \frac{1}{s\theta_1}(a_{61} + a_{12}c\theta_1)$$

and

$$A_6O' = \frac{1}{s\theta_1}(a_{12} + a_{61}c\theta_1).$$

By obtaining analogous results for all six faces, we may write down two alternative expressions for the length of each of six edges, leading to six relations among joint angles, four of which are independent. We may then write, say,

$$s\theta_3(a_{61} + a_{12}c\theta_1) = s\theta_1(a_{34} + a_{23}c\theta_3) \tag{3.1}$$

$$s\theta_5(a_{23} + a_{34}c\theta_3) = s\theta_3(a_{56} + a_{45}c\theta_5) \tag{3.2}$$

$$s\theta_4(a_{12} + a_{23}c\theta_2) = s\theta_2(a_{45} + a_{34}c\theta_4) \tag{3.3}$$

$$s\theta_6(a_{34} + a_{45}c\theta_4) = s\theta_4(a_{61} + a_{56}c\theta_6). \tag{3.4}$$

A fifth independent equation may be obtained by substitution of the dimensional constraints into (A6.9), yielding

$$s\theta_5 s\theta_6 = s\theta_2 s\theta_3. \tag{3.5}$$

Again, several alternative closure equations are available.

Octahedral loops

Bricard set himself the task of solving in part a problem posed by C. Stephanos, namely to establish the existence or non-existence of deformable polyhedra. He shows in [1] that there are three distinct types of octahedra with triangular faces which are fully deformable. One is symmetrical about a line and another about a plane. The third has the property that the "opposite" angles at each vertex are equal or supplementary, and is dubbed [9] by Bricard "doublement aplatissable". Bricard's thorough investigation led to further remarks on the kinematics of the octahedra by Mannheim in the form of a note at the end of [1], and to a considerable extension and geometrical refinement by Bennett [11] who, among other matters, examines chains of mobile octahedra. It is also in this paper that Bennett applies the previously purely geometrical concept of the spherical indicatrix to a jointed assemblage of rigid bodies. Goldberg [12] places the Bricard octahedra into some perspective among polyhedral linkages, and remarks particularly on the property of collapsibility.

Bricard was quick to see the relationship between his octahedra and spatial linkages, and lists [1] the four mobile "hexagones" obtainable from any of the octahedra by removing certain pairs of faces, such as the intersecting pair. Looking at the connection another way, one can view a Bricard octahedron as a very special multiloop linkage in which all eight ternary links have their three revolutes intersecting in pairs. The graph of an octahedron is shown in Fig. 9 which also, coincidentally, points up the duality between an octahedron and a hexahedron.

For all the octahedral linkages,

$$a_{ii+1} = 0, \quad \text{all } i.$$

In order to relate the skew angles between adjacent joints directly to the appropriate angles of the triangular plates, shown in the models to be illustrated later, we adopt a special sign convention. We make the joint offsets alternately positive and negative so that, at every point of intersection, either both offsets are directed towards the point or both directed away from it, as illustrated in Fig. 10. Since each common normal is of zero length, we are free to choose a convenient direction for it. At every point of intersection of offsets, then, we choose its direction so that the usual right-hand screw convention makes the skew angle equal to the angle in the triangle which supports the relevant joint axes. This procedure is also evident in Fig. 10. In measuring joint angles on the models, it is consequently necessary to take some care to observe the appropriate convention, since the common normals will be manifested only as directions.

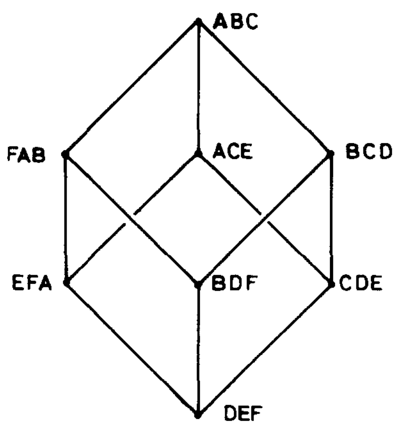

Figure 9.

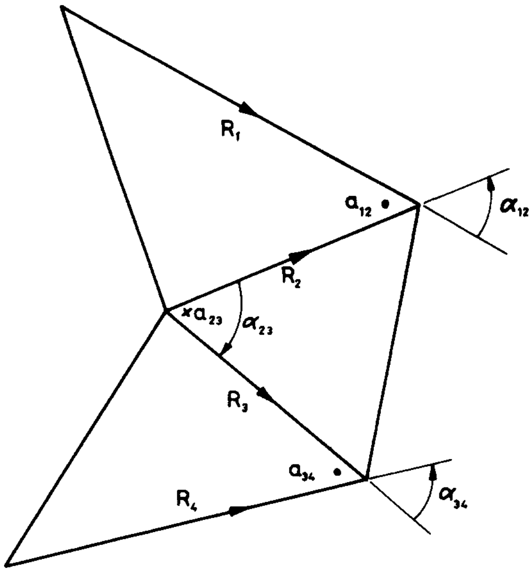

Figure 10.

The line-symmetric octahedral case

Two views of a line-symmetric octahedral linkage model are shown in Figs. 11 and 12. Two views of another model, illustrating an alternative loop from the same octahedron are depicted in Figs. 13 and 14. Figure 15 shows a model due to Goldberg[2] which illustrates a special case of this linkage type, although Goldberg seems, erroneously, to regard it as a trihedral linkage. This latter model is meant to demonstrate that a judicious choice of plate shape will permit complete physical relative movement of the links.

Bricard correctly recognises[9] this linkage as a special case of the general line-symmetric loop. Because of our particular sign convention, however, the constraints on the offsets must take the form

$$R_1+R_4=R_2+R_5=R_3+R_6=0,$$

and the closure eqns (1.1–1.3) must be replaced, respectively, by

$$\theta_1+\theta_4=2\pi \tag{1.1'}$$

$$\theta_2+\theta_5=2\pi \tag{1.2'}$$

$$\theta_3+\theta_6=2\pi. \tag{1.3'}$$

As a result, (1.4) must be replaced by

$$\begin{aligned}&c\alpha_{34}(s\theta_2c\theta_3s\alpha_{12}+c\theta_2s\theta_3s\alpha_{12}c\alpha_{23}+s\theta_3c\alpha_{12}s\alpha_{23})\\&+s\alpha_{34}(s\theta_1c\theta_2c\theta_3+c\theta_1s\theta_2c\theta_3c\alpha_{12}-s\theta_1s\theta_2s\theta_3c\alpha_{23}+c\theta_1c\theta_2s\theta_3c\alpha_{12}c\alpha_{23}\\&-c\theta_1s\theta_3s\alpha_{12}s\alpha_{23})+c\theta_1s\theta_2s\alpha_{23}\\&+s\theta_1c\theta_2c\alpha_{12}s\alpha_{23}+s\theta_1s\alpha_{12}c\alpha_{23}=0,\end{aligned} \tag{1.4'}$$

and (1.5) simplified to yield

$$\begin{aligned}&R_1(s\theta_2c\theta_3s\alpha_{12}+c\theta_2s\theta_3s\alpha_{12}c\alpha_{23}+s\theta_3c\alpha_{12}s\alpha_{23})\\&+R_2(s\theta_1s\alpha_{12}+s\theta_3s\alpha_{23})+R_3(c\theta_1s\theta_2s\alpha_{23}+s\theta_1c\theta_2c\alpha_{12}s\alpha_{23}+s\theta_1s\alpha_{12}c\alpha_{23})\\&=0.\end{aligned} \tag{1.5'}$$

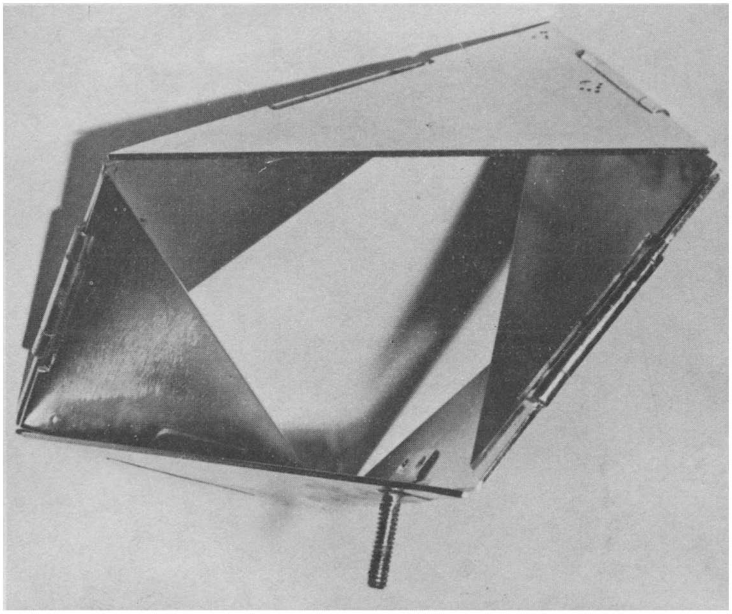

Figure 11.

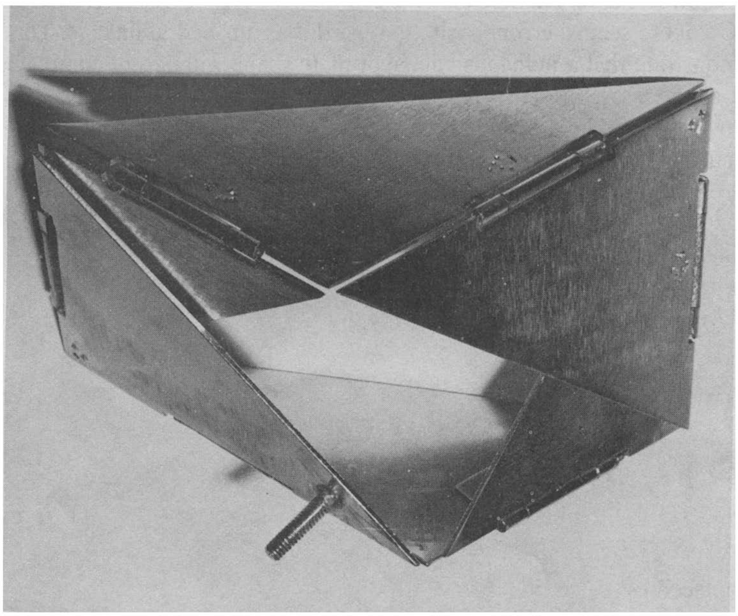

Figure 12.

The above five closure equations have been verified by a series of readings on the two models of Figs. 11–14. The readings, and details of construction of the models, are available in [13]. They are not presented here because their only purpose is verification. As stated earlier, mobility of the octahedral linkages was established in [1].

4. *The plane-symmetric octahedral case*

Although the linkage treated in this subsection is derived from the plane-symmetric octahedron, it is not itself plane-symmetric. Bricard[9] wrongly regarded it as a special case of the loop treated under 2. above, and it seems likely that he was confused over the difference

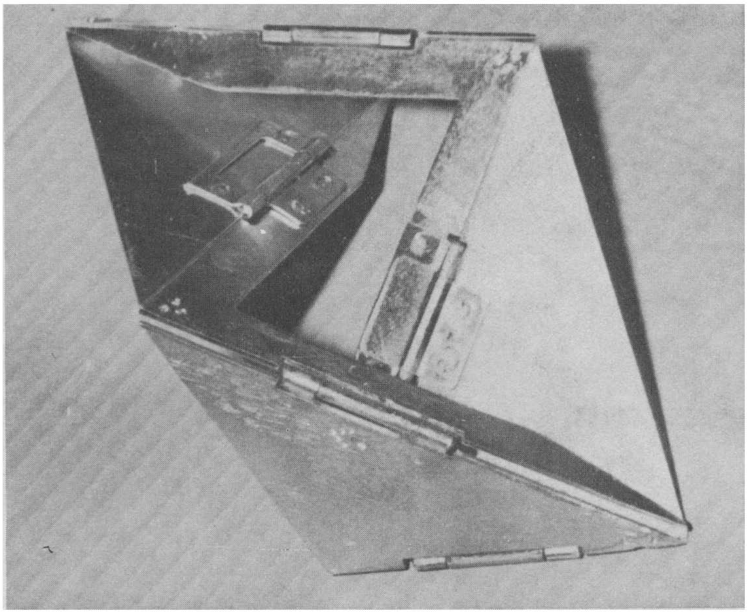

Figure 13.

between link lengths and joint offsets. Considering the period in which he wrote, perhaps his error was not unusual; as mentioned in [6], the concept of joint offset was not common in the literature of the time.

A model of this linkage is illustrated by the two views shown in Figs. 16 and 17. Some elementary geometry and trigonometry applied to the plane-symmetric octahedron yields, for the linkage, under the special sign convention adopted above, the dimensional constraints, say,

$$R_4 = -R_1$$

$$R_2 = -R_1\frac{s\alpha_{34}}{s(\alpha_{12}+\alpha_{34})} \qquad R_5 = R_1\frac{s\alpha_{61}}{s(\alpha_{45}+\alpha_{61})}$$

$$R_3 = R_1\frac{s\alpha_{12}}{s(\alpha_{12}+\alpha_{34})} \qquad R_6 = -R_1\frac{s\alpha_{45}}{s(\alpha_{45}+\alpha_{61})}$$

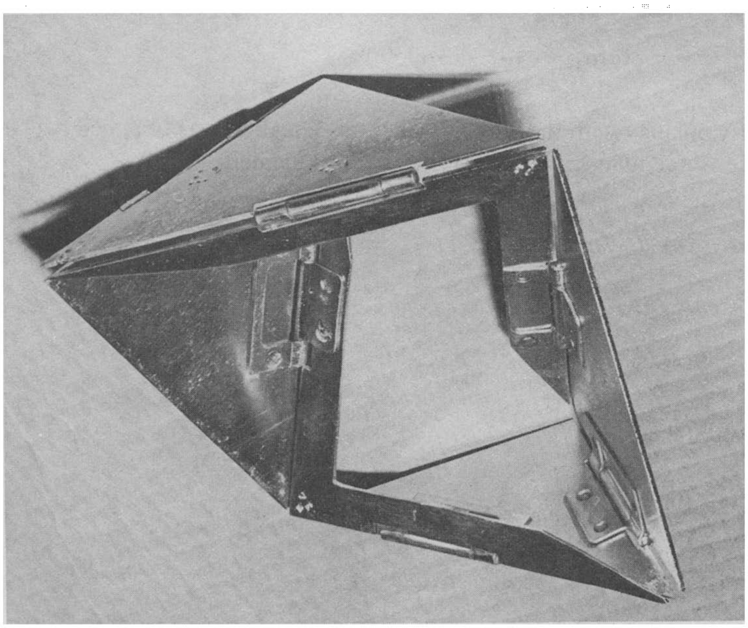

Figure 14.

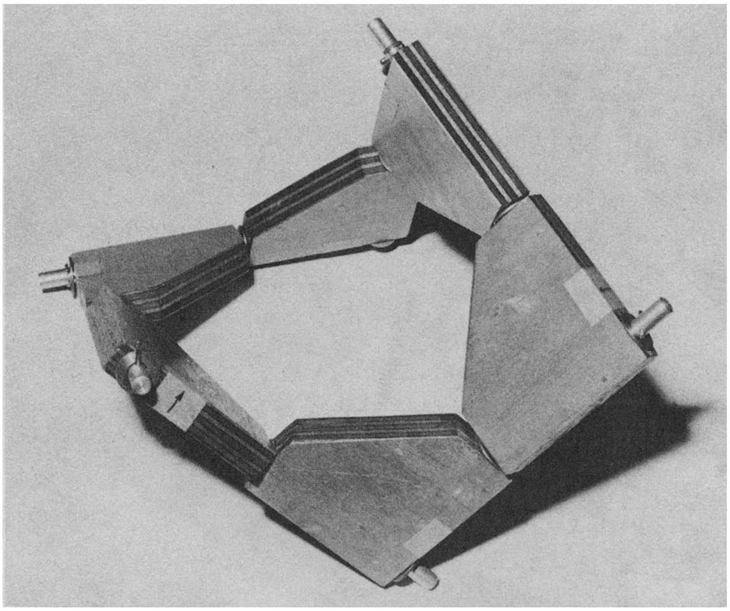

Figure 15.

and the corresponding closure equation,

$$\theta_1 + \theta_4 = 2\pi. \tag{4.1}$$

Substitution of the parametric constraints and (4.1) into (A6.12) with subscripts advanced by 5 yields the result

$$\begin{aligned}\frac{s\alpha_{61}}{s(\alpha_{45}+\alpha_{61})}&(s\alpha_{23}s\alpha_{45}[s\theta_1 s\theta_2 - c\theta_1 c\theta_2 c\alpha_{12}]\\ &+ c\theta_2 s\alpha_{12} s\alpha_{23} c\alpha_{45} + c\theta_1 s\alpha_{45}[s\alpha_{34} - s\alpha_{12} c\alpha_{23}]\\ &- c\alpha_{45}[c\alpha_{12} c\alpha_{23} + c\alpha_{34}])\\ &+ \frac{s\alpha_{34}}{s(\alpha_{12}+\alpha_{34})}(c[\alpha_{12}+\alpha_{34}] + c\alpha_{23}) = 0,\end{aligned} \tag{4.2}$$

after some manipulation. By analogy with the way in which this equation was obtained, we are able to write down a further three independent closure equations, namely

$$\begin{aligned}-\frac{s\alpha_{34}}{s(\alpha_{12}+\alpha_{34})}&(s\alpha_{12}s\alpha_{56}[s\theta_1 s\theta_5 + c\theta_1 c\theta_5 c\alpha_{45}]\\ &- c\theta_5 s\alpha_{45} s\alpha_{56} c\alpha_{12} + c\theta_1 s\alpha_{12}[s\alpha_{45} c\alpha_{56} - s\alpha_{61}]\\ &+ c\alpha_{12}[c\alpha_{61} + c\alpha_{45} c\alpha_{56}])\\ +\frac{s\alpha_{61}}{s(\alpha_{45}+\alpha_{61})}&(c[\alpha_{45}+\alpha_{61}] + c\alpha_{56}) = 0\end{aligned} \tag{4.3}$$

$$\begin{aligned}\frac{s\alpha_{12}}{s(\alpha_{12}+\alpha_{34})}&(s\alpha_{34}s\alpha_{56}[s\theta_1 s\theta_6 - c\theta_1 c\theta_6 c\alpha_{61}] + c\theta_6 s\alpha_{56} s\alpha_{61} c\alpha_{34}\\ &+ c\theta_1 s\alpha_{34}[s\alpha_{45} - s\alpha_{61} c\alpha_{56}] - c\alpha_{34}[c\alpha_{56} c\alpha_{61} + c\alpha_{45}])\\ &+ \frac{s\alpha_{45}}{s(\alpha_{45}+\alpha_{61})}(c[\alpha_{45}+\alpha_{61}] + c\alpha_{56}) = 0\end{aligned} \tag{4.4}$$

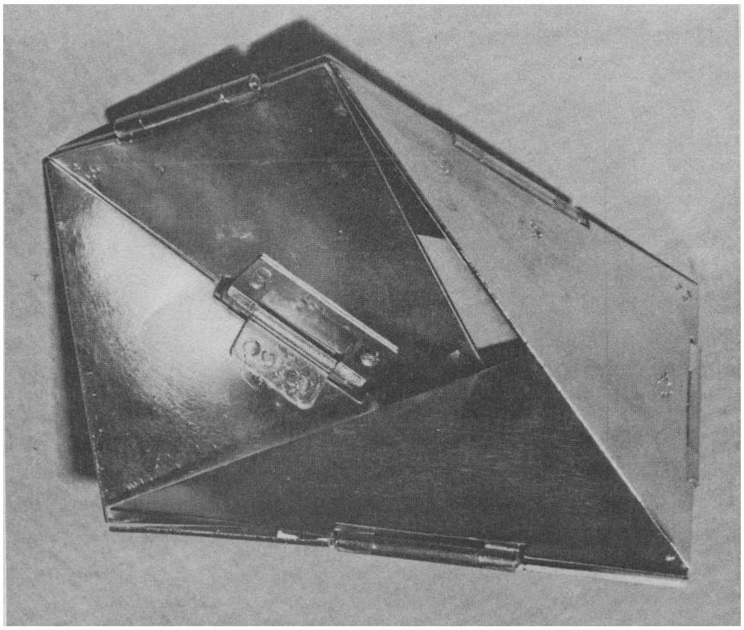

Figure 16.

$$\frac{-s\alpha_{45}}{s(\alpha_{45}+\alpha_{61})}(s\alpha_{23}s\alpha_{61}[s\theta_1 s\theta_3 + c\theta_1 c\theta_3 c\alpha_{34}] - c\theta_3 s\alpha_{23} s\alpha_{34} c\alpha_{61}$$

$$+ c\theta_1 s\alpha_{61}[s\alpha_{34}c\alpha_{23} - s\alpha_{12}] + c\alpha_{61}[c\alpha_{12} + c\alpha_{23}c\alpha_{45}])$$

$$+\frac{s\alpha_{12}}{s(\alpha_{12}+\alpha_{34})}(c[\alpha_{12}+\alpha_{34}] + c\alpha_{23}) = 0. \tag{4.5}$$

Equations (4.1)–(4.5) have been verified by means of a series of readings taken on the model of Figs. 16, 17. Again, detailed results are available in [13]. It is noted that no length term appears explicitly in any of the five equations.

5. *The doubly collapsible octahedral case*

Bennett preferred[11] to describe the parent octahedron of this type as "skew". Certainly,

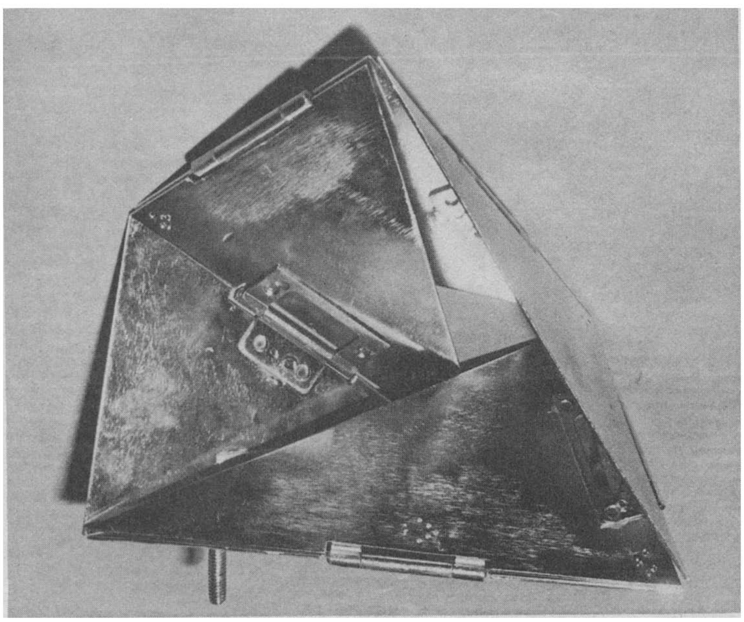

Figure 17.

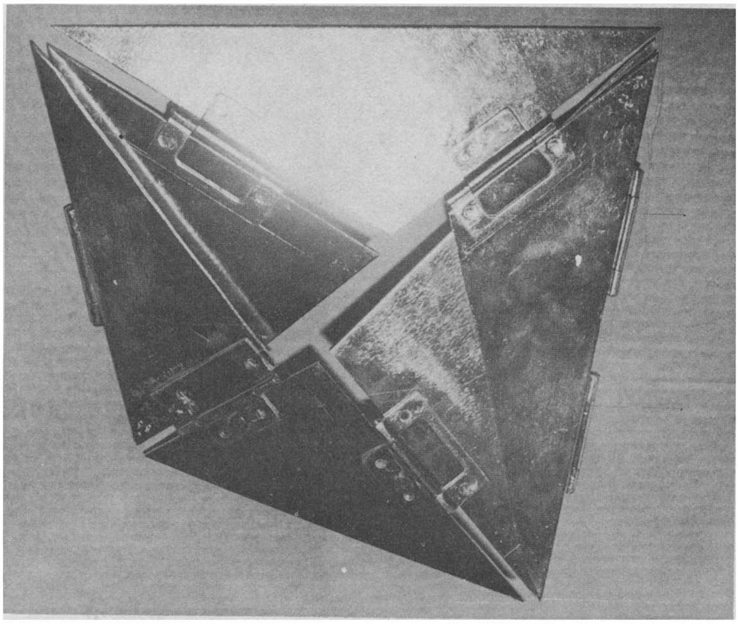

Figure 18.

by comparison with the other two, its overall appearance offers nothing special. Three configurations (two of them collapsed) of a model of this type of loop are shown in Figs. 18–20. The particular property cited above concerning opposing vertical angles results in surprisingly simplified closure equations.

Laid out in schematic form in Fig. 21 are the eight triangles which make up the octahedron. For any pair (β_i, β'_i) of angles, we have the property

$$\beta'_i = \sigma\beta_i + \frac{1-\sigma}{2}\pi, \sigma = \pm 1.$$

Additionally, at any octahedron vertex, both (β_i, β'_i) pairs take the same value of σ. Because the three angles of a triangle sum to π, the number of independent angles among the 24 shown is 4. This result sets the doubly collapsible octahedron apart from the other two, in each of which

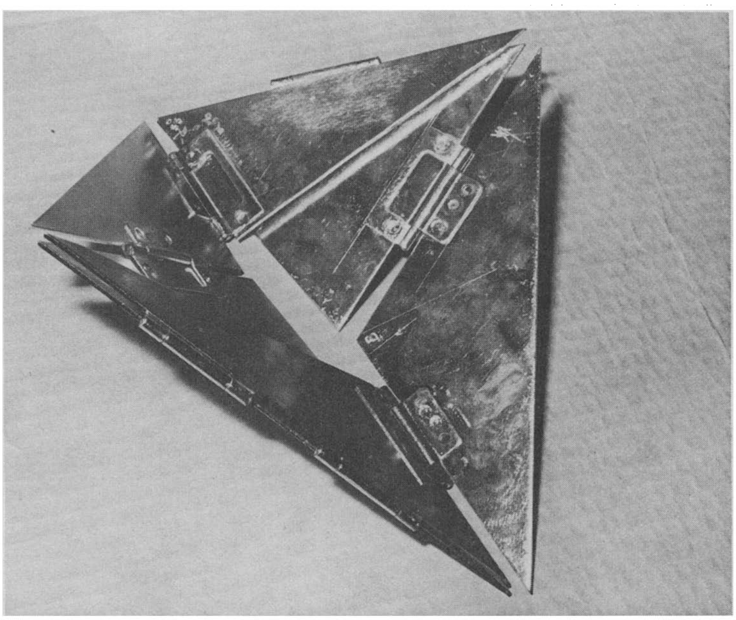

Figure 19.

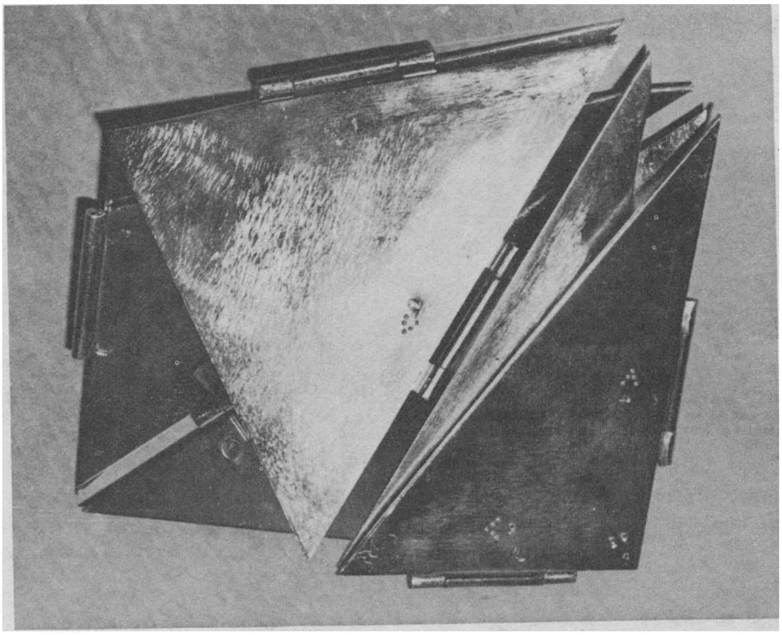

Figure 20.

the number of independent angles is 8. The unusual nature of the constraints, however, prevents us from easily writing down expressions for all angles in terms of four fundamental ones.

To determine the closure equations, we choose for definiteness one of the four possible loops, namely *ABFDEC*. We accordingly designate the offsets as shown in Fig. 21 and find that

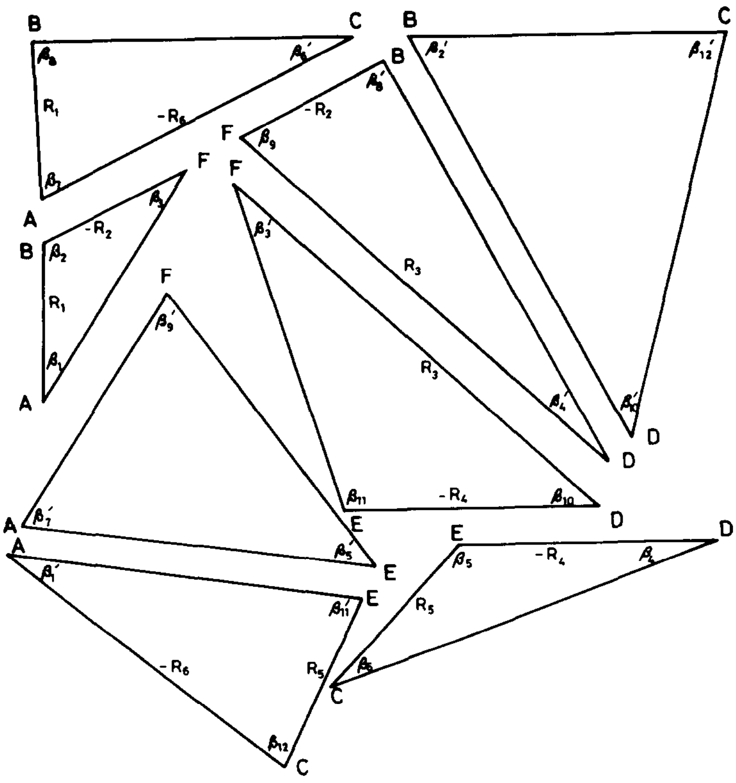

Figure 21.

$$R_2 = -\frac{s\beta_1}{s\beta_3}R_1 \qquad R_3 = \frac{s\beta_8' \, s\beta_1}{s\beta_4' \, s\beta_3}R_1$$

$$R_4 = -\frac{s\beta_8 \, s\beta_1}{s\beta_4 \, s\beta_{11}}R_1$$

$$R_6 = -\frac{s\beta_8}{s\beta_6'}R_1 \qquad R_5 = \frac{s\beta_1' \, s\beta_8}{s\beta_{11}' \, s\beta_6'}R_1,$$

whence

$$R_1R_3R_5 + R_2R_4R_6 = 0.$$

Although we do not use these parametric relationships to find the closure equations, it is worth noting that the offsets are interdependent.

We now pass to Fig. 22, a schematic of the octahedron and the loop which we have chosen. We take advantage of the fact that there are four joint axes passing through each vertex, so that the four corresponding plates alone form a spherical linkage. (In fact, each of the Bricard octahedra may be seen as consisting of six superimposed spherical linkages.) We may therefore apply eqn (A4.9) to vertex A to obtain

$$-c\theta_1 s\beta_1 s\beta_7 + c\beta_1 c\beta_7 = -c\theta_{AE} s\beta_1' s\beta_7' + c\beta_1' c\beta_7',$$

whence

$$c\theta_1 = c\theta_{AE}.$$

We may equally apply (A4.9) to vertex E, noting the conventional direction of R_{AE}, to obtain

$$-c\theta_4 s\beta_5 s\beta_{11} + c\beta_5 c\beta_{11} = -c\theta_{AE} s(\pi - \beta_5')s(\pi - \beta_{11}') + c(\pi - \beta_5')c(\pi - \beta_{11}'),$$

whence

$$c\theta_4 = c\theta_{AE}.$$

From these two results,

$$c\theta_1 = c\theta_4$$

so that, because of our sign convention, we may conclude

$$\theta_1 + \theta_4 = 2\pi. \tag{5.1}$$

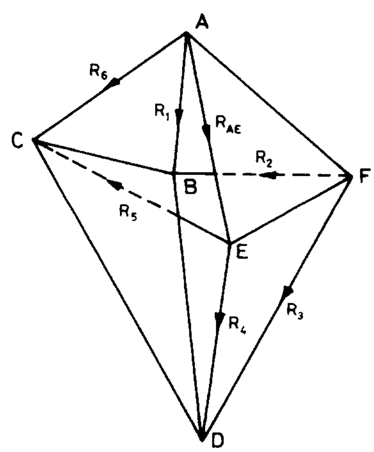

Figure 22.

By analogy, we may also write

$$\theta_2 + \theta_5 = 2\pi \tag{5.2}$$

$$\theta_3 + \theta_6 = 2\pi. \tag{5.3}$$

Again, there is a variety of relationships available to be used as the remaining two necessary independent closure equations. We choose those obtained from (A6.9) by advancing the indices by 1 and 2, respectively. They are

$$\begin{aligned} & s\theta_6 s\theta_1(s\alpha_{56}s\alpha_{12} - s\alpha_{23}s\alpha_{45}) - c\theta_6 c\theta_1(s\alpha_{56}c\alpha_{61}s\alpha_{12} - s\alpha_{23}c\alpha_{34}s\alpha_{45}) \\ & - c\theta_1(c\alpha_{56}s\alpha_{61}s\alpha_{12} - c\alpha_{23}s\alpha_{34}s\alpha_{45}) \\ & - c\theta_6(s\alpha_{56}s\alpha_{61}c\alpha_{12} - s\alpha_{23}s\alpha_{34}c\alpha_{45}) \\ & + (c\alpha_{56}c\alpha_{61}c\alpha_{12} - c\alpha_{23}c\alpha_{34}c\alpha_{45}) = 0 \end{aligned} \tag{5.4}$$

$$\begin{aligned} & s\theta_1 s\theta_2(s\alpha_{61}s\alpha_{23} - s\alpha_{34}s\alpha_{56}) - c\theta_1 c\theta_2(s\alpha_{61}c\alpha_{12}s\alpha_{23} - s\alpha_{34}c\alpha_{45}s\alpha_{56}) \\ & - c\theta_2(c\alpha_{61}s\alpha_{12}s\alpha_{23} - c\alpha_{34}s\alpha_{45}s\alpha_{56}) \\ & - c\theta_1(s\alpha_{61}s\alpha_{12}c\alpha_{23} - s\alpha_{34}s\alpha_{45}c\alpha_{56}) \\ & + (c\alpha_{61}c\alpha_{12}c\alpha_{23} - c\alpha_{34}c\alpha_{45}c\alpha_{56}) = 0. \end{aligned} \tag{5.5}$$

Equations (5.1–5.5), all of which are independent of the joint offsets, have been verified by readings[13] taken on the model of Figs. 18–20.

Closing Remarks

Whilst some of the geometrical aspects of the Bricard linkages were touched upon in the foregoing, much was left unsaid, partly because the paper was already lengthy. There is certainly a good deal which can be investigated from the viewpoint of line geometry and screw system theory, in such matters as axodes, linear complexes and reciprocal screws. Of special interest would be an extension of such research to the original octahedra themselves, but, in this context, Bennett's[11] work should first be studied. It could also be of some value to apply the methods of graph theory, augmented by screw system analysis, to the octahedra, viewing them as multiloop linkages (Fig. 9). For the ambitious, an emulation of Bricard's success for other "concave" polyhedra might be in order, in which case Goldberg's recent paper[14] on the general subject of mobile polyhedra would be recommended reading. It is worth pausing, on this point, to consider the potential hazards in some of the architectural wonders which abound today; the very symmetries and other artistic properties which make them pleasing to the eye might also result in their unexpected structural instability!

In any case, it would certainly be a pity if Bricard's great work were allowed to remain largely unknown for another fifty years, especially when it seems relevant to several areas of study.

Acknowledgements—I am indebted to Dr. K. J. Waldron for the free use of his unpublished notes on the trihedral linkage, which notes provided the basis for the analysis presented in 3. above. I am also grateful to Mr. E. B. T. The, who constructed the octahedral linkage models illustrated herein and who verified the appropriate closure equations developed above by measurement.

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Appendix

Following Waldron[8], we may write down loop closure equations for 6-bar, 5-bar and 4-bar linkages, respectively, in the following form. It is always possible to obtain alternative equations by advancing subscripts, provided the parametric constraints are appropriately observed.

$$\begin{aligned}
&c\theta_4c\theta_5c\theta_6 - s\theta_4s\theta_5c\theta_6c\alpha_{45} - c\theta_4s\theta_5s\theta_6c\alpha_{56} - s\theta_4c\theta_5s\theta_6c\alpha_{45}c\alpha_{56}\\
&+ s\theta_4s\theta_6s\alpha_{45}s\alpha_{56}\\
&= c\theta_1c\theta_2c\theta_3 - s\theta_1s\theta_2c\theta_3c\alpha_{12} - c\theta_1s\theta_2s\theta_3c\alpha_{23} - s\theta_1c\theta_2s\theta_3c\alpha_{12}c\alpha_{23}\\
&+ s\theta_1s\theta_3s\alpha_{12}s\alpha_{23}
\end{aligned} \tag{A6.1}$$

$$\begin{aligned}
&- c\theta_4c\theta_5s\theta_6c\alpha_{61} + s\theta_4s\theta_5s\theta_6c\alpha_{45}c\alpha_{61} - c\theta_4s\theta_5c\theta_6c\alpha_{56}c\alpha_{61}\\
&- s\theta_4c\theta_5c\theta_6c\alpha_{45}c\alpha_{56}c\alpha_{61} + s\theta_4c\theta_5s\alpha_{45}s\alpha_{56}c\alpha_{61} + c\theta_4s\theta_5s\alpha_{56}s\alpha_{61}\\
&+ s\theta_4c\theta_5c\alpha_{45}s\alpha_{56}s\alpha_{61} + s\theta_4s\alpha_{45}c\alpha_{56}s\alpha_{61}\\
&= s\theta_1c\theta_2c\theta_3 + c\theta_1s\theta_2c\theta_3c\alpha_{12} - s\theta_1s\theta_2s\theta_3c\alpha_{23} + c\theta_1c\theta_2s\theta_3c\alpha_{12}c\alpha_{23}\\
&- c\theta_1s\theta_3s\alpha_{12}s\alpha_{23}
\end{aligned} \tag{A6.2}$$

$$\begin{aligned}
&c\theta_4c\theta_5s\theta_6s\alpha_{61} - s\theta_4s\theta_5s\theta_6c\alpha_{45}s\alpha_{61} + c\theta_4s\theta_5c\theta_6c\alpha_{56}s\alpha_{61}\\
&+ s\theta_4c\theta_5c\theta_6c\alpha_{45}c\alpha_{56}s\alpha_{61} - s\theta_4c\theta_6s\alpha_{45}s\alpha_{56}s\alpha_{61} + c\theta_4s\theta_5s\alpha_{56}c\alpha_{61}\\
&+ s\theta_4c\theta_5c\alpha_{45}s\alpha_{56}c\alpha_{61} + s\theta_4s\alpha_{45}c\alpha_{56}c\alpha_{61}\\
&= s\theta_2c\theta_3s\alpha_{12} + c\theta_2s\theta_3s\alpha_{12}c\alpha_{23} + s\theta_3c\alpha_{12}s\alpha_{23}
\end{aligned} \tag{A6.3}$$

$$\begin{aligned}
&s\theta_4c\theta_5c\theta_6 + c\theta_4s\theta_5c\theta_6c\alpha_{45} - s\theta_4s\theta_5s\theta_6c\alpha_{56} + c\theta_4c\theta_5s\theta_6c\alpha_{45}c\alpha_{56}\\
&- c\theta_4s\theta_6s\alpha_{45}s\alpha_{56}\\
&= -c\theta_1c\theta_2s\theta_3c\alpha_{34} + s\theta_1s\theta_2s\theta_3c\alpha_{12}c\alpha_{34} - c\theta_1s\theta_2c\theta_3c\alpha_{23}c\alpha_{34}\\
&- s\theta_1c\theta_2c\theta_3c\alpha_{12}c\alpha_{23}c\alpha_{34} + s\theta_1c\theta_3s\alpha_{12}s\alpha_{23}c\alpha_{34} + c\theta_1s\theta_2s\alpha_{23}s\alpha_{34}\\
&+ s\theta_1c\theta_2c\alpha_{12}s\alpha_{23}s\alpha_{34} + s\theta_1s\alpha_{12}c\alpha_{23}s\alpha_{34}
\end{aligned} \tag{A6.4}$$

$$\begin{aligned}
&- s\theta_4c\theta_5s\theta_6c\alpha_{61} - c\theta_4s\theta_5s\theta_6c\alpha_{45}c\alpha_{61} - s\theta_4s\theta_5c\theta_6c\alpha_{56}c\alpha_{61}\\
&+ c\theta_4c\theta_5c\theta_6c\alpha_{45}c\alpha_{56}c\alpha_{61} - c\theta_4c\theta_6s\alpha_{45}s\alpha_{56}c\alpha_{61} + s\theta_4s\theta_5s\alpha_{56}s\alpha_{61}\\
&- c\theta_4c\theta_5c\alpha_{45}s\alpha_{56}s\alpha_{61} - c\theta_4s\alpha_{45}c\alpha_{56}s\alpha_{61}\\
&= -s\theta_1c\theta_2s\theta_3c\alpha_{34} - c\theta_1s\theta_2s\theta_3c\alpha_{12}c\alpha_{34} - s\theta_1s\theta_2c\theta_3c\alpha_{23}c\alpha_{34}\\
&+ c\theta_1c\theta_2c\theta_3c\alpha_{12}c\alpha_{23}c\alpha_{34} - c\theta_1c\theta_3s\alpha_{12}s\alpha_{23}c\alpha_{34} + s\theta_1s\theta_2s\alpha_{23}s\alpha_{34}\\
&- c\theta_1c\theta_2c\alpha_{12}s\alpha_{23}s\alpha_{34} - c\theta_1s\alpha_{12}c\alpha_{23}s\alpha_{34}
\end{aligned} \tag{A6.5}$$

$$\begin{aligned}
&s\theta_4c\theta_5s\theta_6s\alpha_{61} + c\theta_4s\theta_5s\theta_6c\alpha_{45}s\alpha_{61} + s\theta_4s\theta_5c\theta_6c\alpha_{56}s\alpha_{61}\\
&- c\theta_4c\theta_5c\theta_6c\alpha_{45}c\alpha_{56}s\alpha_{61} + c\theta_4c\theta_6s\alpha_{45}s\alpha_{56}s\alpha_{61} + s\theta_4s\theta_5s\alpha_{56}c\alpha_{61}\\
&- c\theta_4c\theta_5c\alpha_{45}s\alpha_{56}c\alpha_{61} - c\theta_4s\alpha_{45}c\alpha_{56}c\alpha_{61}\\
&= -s\theta_2s\theta_3s\alpha_{12}c\alpha_{34} + c\theta_2c\theta_3s\alpha_{12}c\alpha_{23}c\alpha_{34} + c\theta_3c\alpha_{12}s\alpha_{23}c\alpha_{34}\\
&- c\theta_2s\alpha_{12}s\alpha_{23}s\alpha_{34} + c\alpha_{12}c\alpha_{23}s\alpha_{34}
\end{aligned} \tag{A6.6}$$

$$\begin{aligned}
&s\theta_5c\theta_6s\alpha_{45} + c\theta_5s\theta_6s\alpha_{45}c\alpha_{56} + s\theta_6c\alpha_{45}s\alpha_{56}\\
&= c\theta_1c\theta_2s\theta_3s\alpha_{34} - s\theta_1s\theta_2s\theta_3c\alpha_{12}s\alpha_{34} + c\theta_1s\theta_2c\theta_3c\alpha_{23}s\alpha_{34}\\
&+ s\theta_1c\theta_2c\theta_3c\alpha_{12}c\alpha_{23}s\alpha_{34} - s\theta_1c\theta_3s\alpha_{12}s\alpha_{23}s\alpha_{34} + c\theta_1s\theta_2s\alpha_{23}c\alpha_{34}\\
&+ s\theta_1c\theta_2c\alpha_{12}s\alpha_{23}c\alpha_{34} + s\theta_1s\alpha_{12}c\alpha_{23}c\alpha_{34}
\end{aligned} \tag{A6.7}$$

$$-s\theta_5 s\theta_6 s\alpha_{45} c\alpha_{61} + c\theta_5 c\theta_6 s\alpha_{45} c\alpha_{56} c\alpha_{61} + c\theta_6 c\alpha_{45} s\alpha_{56} c\alpha_{61}$$
$$-c\theta_5 s\alpha_{45} s\alpha_{56} s\alpha_{61} + c\alpha_{45} c\alpha_{56} s\alpha_{61}$$
$$= s\theta_1 c\theta_2 s\theta_3 s\alpha_{34} + c\theta_1 s\theta_2 s\theta_3 c\alpha_{12} s\alpha_{34} + s\theta_1 s\theta_2 c\theta_3 c\alpha_{23} s\alpha_{34}$$
$$-c\theta_1 c\theta_2 c\theta_3 c\alpha_{12} c\alpha_{23} s\alpha_{34} + c\theta_1 c\theta_3 s\alpha_{12} s\alpha_{23} s\alpha_{34} + s\theta_1 s\theta_2 s\alpha_{23} c\alpha_{34}$$
$$-c\theta_1 c\theta_2 c\alpha_{12} s\alpha_{23} c\alpha_{34} - c\theta_1 s\alpha_{12} c\alpha_{23} c\alpha_{34} \tag{A6.8}$$

$$s\theta_5 s\theta_6 s\alpha_{45} s\alpha_{61} - c\theta_5 c\theta_6 s\alpha_{45} c\alpha_{56} s\alpha_{61} - c\theta_6 c\alpha_{45} s\alpha_{56} s\alpha_{61}$$
$$-c\theta_5 s\alpha_{45} s\alpha_{56} c\alpha_{61} + c\alpha_{45} c\alpha_{56} c\alpha_{61}$$
$$= s\theta_2 s\theta_3 s\alpha_{12} s\alpha_{34} - c\theta_2 c\theta_3 s\alpha_{12} c\alpha_{23} s\alpha_{34} - c\theta_3 c\alpha_{12} s\alpha_{23} s\alpha_{34}$$
$$-c\theta_2 s\alpha_{12} s\alpha_{23} c\alpha_{34} + c\alpha_{12} c\alpha_{23} c\alpha_{34} \tag{A6.9}$$

$$a_{12}(c\theta_2\theta_3 - s\theta_2 s\theta_3 c\alpha_{23}) + r_1(s\theta_2 c\theta_3 s\alpha_{12} + c\theta_2 s\theta_3 s\alpha_{12} c\alpha_{23} + s\theta_3 c\alpha_{12} s\alpha_{23})$$
$$+ a_{23} c\theta_3 + r_2 s\theta_3 s\alpha_{23} + a_{34} + a_{45} c\theta_4$$
$$+ a_{56}(c\theta_4 c\theta_5 - s\theta_4 s\theta_5 c\alpha_{45}) + r_5 s\theta_4 s\alpha_{45}$$
$$+ a_{61}(c\theta_4 c\theta_5 c\theta_6 - s\theta_4 s\theta_5 c\theta_6 c\alpha_{45} - c\theta_4 s\theta_5 s\theta_6 c\alpha_{56} - s\theta_4 c\theta_5 s\theta_6 c\alpha_{45} c\alpha_{56} + s\theta_4 s\theta_6 s\alpha_{45} s\alpha_{56})$$
$$+ r_6(c\theta_4 s\theta_5 s\alpha_{56} + s\theta_4 c\theta_5 c\alpha_{45} s\alpha_{56} + s\theta_4 s\alpha_{45} c\alpha_{56})$$
$$= 0 \tag{A6.10}$$

$$a_{12}(-c\theta_2 s\theta_3 c\alpha_{34} - s\theta_2 c\theta_3 c\alpha_{23} c\alpha_{34} + s\theta_2 s\alpha_{23} s\alpha_{34})$$
$$+ r_1(-s\theta_2 s\theta_3 s\alpha_{12} c\alpha_{34} + c\theta_2 c\theta_3 s\alpha_{12} c\alpha_{23} c\alpha_{34} + c\theta_3 c\alpha_{12} s\alpha_{23} c\alpha_{34} - c\theta_2 s\alpha_{12} s\alpha_{23} s\alpha_{34} + c\alpha_{12} c\alpha_{23} s\alpha_{34})$$
$$- a_{23}\theta_3 c\alpha_{34} + r_2(c\theta_3 s\alpha_{23} c\alpha_{34} + c\alpha_{23} s\alpha_{34}) + r_3 s\alpha_{34}$$
$$+ a_{45} s\theta_4 + a_{56}(s\theta_4 c\theta_5 + c\theta_4 s\theta_5 c\alpha_{45}) - r_5 c\theta_4 s\alpha_{45}$$
$$+ a_{61}(s\theta_4 c\theta_5 c\theta_6 + c\theta_4 s\theta_5 c\theta_6 c\alpha_{45} - s\theta_4 s\theta_5 s\theta_6 c\alpha_{56} + c\theta_4 c\theta_5 s\theta_6 c\alpha_{45} c\alpha_{56} - c\theta_4 s\theta_6 s\alpha_{45} s\alpha_{56})$$
$$+ r_6(s\theta_4 s\theta_5 s\alpha_{56} - c\theta_4 c\theta_5 c\alpha_{45} s\alpha_{56} - c\theta_4 s\alpha_{45} c\alpha_{56})$$
$$= 0 \tag{A6.11}$$

$$a_{12}(c\theta_2 s\theta_3 s\alpha_{34} + s\theta_2 c\theta_3 c\alpha_{23} s\alpha_{34} + s\theta_2 s\alpha_{23} c\alpha_{34}$$
$$+ r_1(s\theta_2 s\theta_3 s\alpha_{12} s\alpha_{34} - c\theta_2 c\theta_3 s\alpha_{12} c\alpha_{23} s\alpha_{34} - c\theta_3 c\alpha_{12} s\alpha_{23} s\alpha_{34} - c\theta_2 s\alpha_{12} s\alpha_{23} c\alpha_{34} + c\alpha_{12} c\alpha_{23} c\alpha_{34})$$
$$+ a_{23} s\theta_3 s\alpha_{34} + r_2(c\alpha_{23} c\alpha_{34} - c\theta_3 s\alpha_{23} s\alpha_{34}) + r_3 c\alpha_{34}$$
$$+ r_4 + a_{56} s\theta_5 s\alpha_{45} + r_5 c\alpha_{45} + a_{61}(s\theta_5 c\theta_6 s\alpha_{45} + c\theta_5 s\theta_6 s\alpha_{45} c\alpha_{56} + s\theta_6 c\alpha_{45} s\alpha_{56})$$
$$+ r_6(c\alpha_{45} c\alpha_{56} - c\theta_5 s\alpha_{45} s\alpha_{56}) = 0 \tag{A6.12}$$

$$-c\theta_2 s\alpha_{12} s\alpha_{23} + c\alpha_{12} c\alpha_{23}$$
$$= s\theta_4 s\theta_5 s\alpha_{34} s\alpha_{51} - c\theta_4 c\theta_5 s\alpha_{34} c\alpha_{45} s\alpha_{51} - c\theta_4 s\alpha_{34} s\alpha_{45} c\alpha_{51}$$
$$-c\theta_5 c\alpha_{34} s\alpha_{45} s\alpha_{51} + c\alpha_{34} c\alpha_{45} c\alpha_{51} \tag{A5.9}$$

$$a_{51}(s\theta_4 c\theta_5 s\alpha_{34} + c\theta_4 s\theta_5 s\alpha_{34} c\alpha_{45} + s\theta_5 c\alpha_{34} s\alpha_{45})$$
$$+ r_5(c\alpha_{34} c\alpha_{45} - c\theta_4 s\alpha_{34} s\alpha_{45}) + a_{45} s\theta_4 s\alpha_{34} + r_4 c\alpha_{34} + r_3 + r_2 c\alpha_{23}$$
$$+ a_{12} s\theta_2 s\alpha_{23} + r_1(c\alpha_{12} c\alpha_{23} - c\theta_2 s\alpha_{12} s\alpha_{23}) = 0 \tag{A5.12}$$

$$-c\theta_2 s\alpha_{12} s\alpha_{23} + c\alpha_{12} c\alpha_{23} = -c\theta_4 s\alpha_{34} s\alpha_{45} + c\alpha_{34} c\alpha_{45} \tag{A4.9}$$

UNE ANALYSE DES MECANISMES DE BRICARD

J. Eddie Baker

Résumé - Maintenant les cinématiciens connaissent et comprennent plusieurs mécanismes surcontraints mais, il y a cinquante ans, ils étaient considérés comme "paradoxaux". Parmi les chaînes surcontraintes les plus significatives et les plus intéressantes sont celles publiées au début du siècle par Bricard [1,9]. Bien que d'autres chercheurs [2,7,11-13] aient commenté ces mécanismes et/ou aient amplifié les aspects géométriques de ces octaèdres déformables, aucune publication n'a analysé le mouvement relatif entre les membres. L'objet de cet article est d'utiliser la méthode des équations de clôture [3,6,8] pour décrire pleinement les cinq mécanismes distincts. Cette analyse complète le travail sur les mécanismes surcontraints connus qui ne contiennent que les couples tournants [5,6].

La première chaîne de Bricard est la boucle générale à ligne de symétrie (Fig. 2), conditionnée par les équations (1.1-5). La deuxième est le mécanisme général à plan de symétrie (Fig. 4), qui est sujet aux équations (2.1-5). Le troisième type distinct est le mécanisme trièdre unique (Figs. 7,8), conditionné par les équations (3.1-5). On peut dériver tout droit le restant des trois octaèdres déformables de Bricard [1], mais l'un d'eux est un cas spécial de la chaîne à ligne de symétrie susmentionnée. La quatrième boucle (Figs. 16,17) est dérivée d'un octaèdre à plan de symétrie, mais n'a pas elle-même un plan de symétrie. Elle est conditionnée par les équations (4.1-5). La cinquième chaîne est le mécanisme "doublement aplatissable" (Figs. 18-20), sujet aux équations (5.1-5).

Les équations de clôture (A6.1-12) du mécanisme général à six membres sont très difficiles à appliquer à cause du nombre et de la complexité des termes dans chaque équation. Dans cet article, on fait des simplifications dans la mesure du possible en appliquant les équations à quatre membres et à cinq membres aux parties appropriées des chaînes à six membres.

On fait quelques suggestions pour les travaux futurs possibles dans le cas de ces mécanismes et des octaèdres connexes.